

OUR ASTRONOMICAL COLUMN.

ASTRONOMICAL OCCURRENCES IN JANUARY, 1905.

- Jan. 2-3. Epoch of January meteors (Radiant $230^{\circ} + 53^{\circ}$).
 6. 4h. 52m. to 7h. 5m. Transit of Jupiter's Sat. III. (Ganymede).
 8. 2h. Saturn in conjunction with Moon (Saturn $3^{\circ} 3' S.$).
 9. 3h. Venus in conjunction with Moon (Venus $2^{\circ} 13' S.$).
 „ 11h. Juno in conjunction with Moon (Juno $0^{\circ} 11' S.$).
 10. 5h. 9m. to 6h. 23m. Moon occults ϕ Aquarii (Mag. 4.4).
 11. Perihelion Passage of Encke's Comet.
 13. 8h. 52m. to 11h. 6m. Transit of Jupiter's Sat. III. (Ganymede).
 „ 10h. 36m. Minimum of Algol (β Persei).
 15. Venus. Illuminated portion of disc = 0.650, of Mars = 0.903.
 16. 7h. 25m. Minimum of Algol (β Persei).
 24. 12h. 43m. to 13h. 40m. Moon occults β Virginis (Mag. 3.8).
 27. 10h. Mars in conjunction with Moon (Mars $2^{\circ} 45' S.$).
 28. 15h. 7m. to 16h. 11m. Moon occults γ Libræ (Mag. 4.1).

ELEMENTS AND EPHEMERIS OF COMET 1904 d.—Circular No. 69 from the Kiel Centralstelle contains a set of elements, calculated by Herr M. Ebell from the observations made on December 17, 18, 19, and a short ephemeris, for comet 1904 d, recently discovered by M. Giacobini at Nice. They are as follows:—

Elements.

$$\begin{aligned} T &= 1905 \text{ Jan. } 3^{\text{h}} 28^{\text{m}} 14^{\text{s}} \text{ Berlin.} \\ \infty &= 75^{\circ} 9' 8'' \\ Q &= 225^{\circ} 1' 2'' \\ i &= 103^{\circ} 27' 3'' \\ \log q &= 0.27173 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} 1904.0$$

1904-5	Ephemeris 12h. (M.T. Berlin).			δ	$\log \Delta$	Bright-ness
	h.	m.	s.			
Dec. 26	16	37	56	+31 45	0.3328	1.12
„ 30	16	49	48	+33 53	0.3234	1.17
Jan. 3	17	2	37	+36 8	0.3146	1.22

Brightness at time of discovery = 1.0.

From the above it will be seen that both the northern declination and the brightness of the comet are increasing, but at the same time its right ascension is approximating more closely to that of the sun, thereby rendering observations increasingly difficult, and only possible during the few minutes preceding dawn.

OBSERVATIONS OF BRIGHT METEORS.—During a sea voyage undertaken in 1903-4, Dr. J. Möller, of Elsfleth, observed a large number of meteors, and in No. 3984 of the *Astronomische Nachrichten* he records the essential data regarding the observations of the sixteen brightest objects seen during November-December, 1903, and March, 1904. Of these, two were as bright as Jupiter, and five were brighter than Saturn. The latitude and longitude of the place of observation are given in each case, so that in the event of duplicate observations having been made the real paths may be computed.

The same observer recorded in No. 3971 of the same journal an authenticated naked-eye observation of Jupiter's third satellite on November 1, 1903.

THE GREAT RED SPOT ON JUPITER.—In a note to No. 3983 of the *Astronomische Nachrichten* Mr. Denning gives the results of his own and the Rev. T. E. Phillips's observations of the Great Red Spot since the last conjunction of Jupiter. They show that for the seven months prior to last September the motion of the spot indicated a rotation period, for the zone wherein it is located, of 9h. 55m. 38.6s., a shorter period than any observed since 1883, when it was 9h. 55m. 38.2s.

In the same publication Mr. Stanley Williams gives the results of his observations of this phenomenon, and shows that from his eye-estimates of the times of transit, during

the period August, 1903, to January, 1904, the average time of rotation was 9h. 55m. 41.52s.

He points out that this is a remarkable increase on the rotation period (viz. 9h. 55m. 39.66s.) of the preceding year.

REPORT OF THE UNITED STATES NAVAL OBSERVATORY.—Rear-Admiral Chester's report of the work done at the United States Naval Observatory during the fiscal year ending June 30, 1904, shows that the observatory and the staff are still maintaining their reputation as regards the number and excellence of the observations made. In all 15,287 observations were made, including photographs of the sun taken on 210 days which show an increase of 93 days on which spots and faculae were recorded on the solar disc. A new photo-visual triple objective with an aperture of 7.5 inch and a focal length of 65 feet, giving a 7-inch image, is to be obtained for the photoheliograph, and will also be used on future eclipse expeditions for photographing the corona. In regard to next year's eclipse the superintendent asks for a special grant of 1200l. and recommends the employment of a man-of-war and its crew to assist in the observations, which he suggests should be made at two widely separated stations in Spain.

The report also contains individual reports from the assistant in charge of each department, and records the personnel, the routine work performed with each instrument, and the publications issued during the period with which it deals.

The branch observatory at Tutuila, Samoa, has now been established, and placed under the supervision of assistants from Washington.

MATHEMATICAL DRAWING.¹

THE appearance of a useful little book by Prof. Gibson may be made the occasion of emphasising the importance of drawing in mathematics, whether pure or applied, especially as the University of London has recently made a paper on drawing compulsory for all mathematical candidates for the B.Sc. degree. It was not without due consideration of the attendant difficulties that this step was taken. For the last two years the paper on drawing was left optional for the candidates in order that teachers as well as students should have time to obtain some definite notion of what is required; but even now, in the absence of well established text-books, a considerable amount of uncertainty exists as to the nature and scope of the subject. Time will, no doubt, set this right, and we welcome Prof. Gibson's text-book as assisting towards the desired object.

There are three prominent conceptions of mathematical drawing which may be noticed. These are:—(1) plotting, which means the construction of curves by taking a set of successive values of an abscissa and from them calculating (by a book of tables or otherwise) the values of the corresponding ordinate, and finally marking the positions of the points on squared paper; (2) the construction of curves—usually conic sections—from certain geometrical data; (3) what is generally called “geometrical drawing,” embodying the principles and processes of projective geometry, and including problems in three dimensions. This is, perhaps, a rough division, but it will suffice.

Plotting may be a very humble process—“mere” plotting, as it is sometimes contemptuously called—or it may be what has long been known as curve tracing, and is to be found in treatises on the differential calculus. But even in this latter and higher character it is not (at least as usually employed by students) a system of accurate drawing. The construction of circles, and conics generally, from assigned data is certainly not a pure exercise in drawing, because it involves a very large knowledge of theorems on the part of the student. An exercise in this subject is apt to be, in reality, a severe examination in Euclid or in the theory of conic sections, and it cannot be what was intended by the advocates of a paper on drawing. With regard to projective geometry the case is somewhat different; the principles involved are not very numerous, and it cannot be said that a

¹ “An Elementary Treatise on Graphs.” By George A. Gibson, M.A., F.R.S.E., Professor of Mathematics in the Glasgow and West of Scotland Technical College. Pp. x + 183. (London: Macmillan and Co., Ltd.) Price 3s. 6d.

knowledge of a large assortment of theorems is necessary; but the practical value of the study to students who are neither engineers nor architects is another matter.

There is, however, another kind of mathematical drawing which does not fall under any of these heads, and which consists in the invention of graphic solutions of equations which can be solved with great difficulty, if at all, by the stock processes of accurate mathematics. This branch is at once the most useful and the most vague; it is impossible to lay down its principles in systematic order—it must be learnt by abundant exemplification.

The ordinary academic problems of statics and hydrostatics furnish many examples of this subject, but only a few of these can be noticed here.

If AB and BC are two ladders freely jointed together at B, of different weights and lengths, placed with the ends A and C resting on a rough horizontal plane, A being prevented from moving while C is drawn out along the plane, the inclinations, θ , ϕ , of AB and BC to the ground when the limiting position is reached are determined from two equations of the forms

$$a \sin \theta - b \sin \phi = 0; m \tan \theta + n \tan \phi = k,$$

where a , b , m , n , k are all given quantities. The graphic solution of these equations is effected with great ease thus:—draw a line OH equal to m , and produce OH to O' so that HO' = n ; at H draw HC perpendicular to OO' and equal to k ; through O draw any line OQ meeting HC in Q; take a point R in CH such that CR = HQ, and draw O'R; then the point P, of intersection of OQ and O'R is a point on the locus represented by the second of the above equations, the angles θ , ϕ being POO' and PO'O. These points, P, are therefore constructed with great ease and rapidity. Also the locus represented by the first equation is a circle having for diameter the line joining the points which divide OO' internally and externally in the ratio $a:b$, and the points of intersection of these two loci give the required values of θ and ϕ .

The following problem leads to precisely the same equations as the above:—rays of light emanate from a fixed point P in one medium separated by a plane surface from a second medium; find the ray proceeding from P which will be refracted to a given point, Q, in the second medium.

Again, the fact that when a uniform chain hangs with free extremities over two fixed supports of equal heights there are either two figures of equilibrium or none results from the solution of an equation of the form $xe^{ax}/x = k$, which is effected by drawing the curve $y = e^x$ and the right line $y = kx/a$, and then it is at once seen that there are either two points of intersection or none.

When a heavy wire rope has its ends fixed at two points in the same horizontal line, and a load is suspended from the lowest point of the rope, the rope forms parts of two distinct catenaries, and the determination of these curves leads to an equation of the form

$$e^{k/x} = [(x^2 + a^2)^{\frac{1}{2}} + a] / [(x^2 + b^2)^{\frac{1}{2}} + b],$$

in which x alone is unknown. The tracing of the curve obtained by putting y equal to the right-hand side of this equation is quickly effected by means of two fixed circles and the drawing of right lines.

The figure of equilibrium of a revolving self-attracting liquid spheroid gives an equation which is a particular case of $x(a + bx^2)/(c + x^2) = \tan^{-1}x$, and this is best solved by the tracing of two curves. If we put y equal to the left-hand side we have a curve of the third degree the geometrical construction of which is exceedingly simple, and requires only a fixed circle and right lines.

Whenever a problem involves two unknown angles in two equations one of which is of the form $m \cos \theta + n \cos \phi = c$, where m , n , c are given, all angles satisfying this equation can be represented as the base angles of a triangle the base of which, AB, is fixed, and the vertex of which describes what may be called a quasi-magnetic curve, the geometrical construction of which is this: take any two fixed points, A, B; about A as centre, with radius $m \cdot AB/c$ describe a circle; about B describe a circle with radius $n \cdot AB/c$; draw any line perpendicular to AB meeting these circles in Q and R respectively; then the lines AQ and BR intersect in a point on the required curve. When $m=n$ we have the common magnetic curve the construction of which is not nearly so well known as it should be.

The solutions of the above examples have all been of a purely geometrical kind, and have not involved the plotting of points by coordinates arithmetically calculated. There are other problems of a slightly different kind, still independent of plotting, but involving trial; the value of a certain unknown quantity which has to satisfy a certain geometrical condition is found by trial to do so very nearly if not completely. In all such cases Taylor's theorem furnishes a still closer value than the observed one, and completes the solution with all desirable accuracy.

For example, many problems lead to the equation $a \sin 2(\theta - \alpha) = b \sin \theta$ for an unknown angle θ , the other quantities being all given. This can be solved by two circles thus:—draw a line AB equal to b , and on it as diameter describe a circle the centre of which is C; draw AD making the angle BAD = α and cutting the circle in D; draw CD and produce it to E so that CE = a , and on CE as diameter describe a circle. Now find on the circumference of the first circle a point P such that if CP meets the second circle in Q we have BP = EQ. This is done with great accuracy by the eye, and Taylor's theorem will improve the solution.

An equation which can be solved also very easily by trial is $a \sin^2 \theta = b \cot \theta$, which may be taken in the form $a \sin^3 \theta = b \cos \theta$, and a graphic solution suitable to each form is easily found.

Finally, we may notice equations of the form

$$\tan x = ax/(c - x^2),$$

which we obtain from Bessel functions in certain problems relating to vibrations. Such an equation is easily solved by the intersections of the curve $y = \cot x$ with the hyperbola $y = (c - x^2)/ax$, and the construction of the hyperbola belongs to the most simple case of this curve, viz. given one point on the curve and the asymptotes. As compared with the graphic solution of equations given by physical problems, the graphic solution of algebraic equations is unimportant, though not devoid of interest, because Horner is always available for numerical cases.

Prof. Gibson gives many examples of the solutions of quadratics and of cubics by graphic methods; but as regards quadratics it must be confessed that there is no utility in the process, and too much space is usually devoted to it. For cubics in general he gives a graphic solution and an interesting discussion. In a second edition of his book he might treat the biquadratic similarly, because its graphic solution can be easily effected by means of a circle and a parabola, or by means of a right line and a curve easily derived from a parabola. Many curves occurring in physics are dealt with in the book—such as isothermals and adiabatics; there is also a useful discussion of Fourier's theorem, and a treatment of the curves belonging to vibrations, damped as well as undamped. The graphic method is also applied to the solution of some of the simpler mixed trigonometric and algebraic equations, and the book concludes with a chapter on the properties of conic sections.

GEORGE M. MINCHIN.

CENTRAL AMERICAN MAMMALS.¹

THREE years ago the author of these volumes published, in the same serial, a valuable synopsis of the mammals of North America and the adjacent seas. In the present larger work he has taken in hand the mammals of the tract generally known in this country as Central America, but on the other side of the Atlantic termed, at any rate by zoologists, Middle America, together with those of the West Indian islands. The greater bulk of the present work is accounted for, not so much by the greater number of species (690 against 606) as by the increased elaboration of the mode of treatment, the addition of diagnostic "keys" to the various genera, and by a fuller account of the habits of many species, the latter feature rendering these volumes proportionately more valuable to the naturalist, and at the same time of more general interest. The illustrations, too, are more numerous, comprising, besides crania, figures of the external form of a considerable number of species,

¹ "The Land and Sea Mammals of Middle America and the West Indies." By D. G. Elliot. *Field Columbian Museum Publications*, Zoological Series, vol. iv., part. i. and ii., pp. xxi+850 illustrated.